PB83-145409

Bending of Laminated Anisotropic Shells by a Shear Deformable Finite Element

Virginia Polytechnic Inst. and State Univ.
Blacksburg

Prepared for

Air Force Office of Scientific Research
Bolling AFB, DC

Oct 82

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER VPI-E-82.30	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER PB83 145409
4. TITLE (and Subtitle) BENDING OF LAMINATED ANISOTROPIC SHELLS BY A SHEAR DEFORMABLE FINITE ELEMENT		5. TYPE OF REPORT & PERIOD COVERED Interim
		6. PERFORMING ORG. REPORT NUMBER Tech. Report No. AFOSR-81-4
7. AUTHOR(s) J. N. Reddy		8. CONTRACT OR GRANT NUMBER(s) AFOSR-81-0142
9. PERFORMING ORGANIZATION NAME AND ADDRESS Virginia Polytechnic Institute and State University Blacksburg, Virginia 24061 ESM Department		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS United States Air Force Air Force Office of Scientific Research Bldg. 410, Boling AFB, Washington, D.C. 20332		12. REPORT DATE October, 1982
		13. NUMBER OF PAGES 28
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

This document has been approved for public release and sale; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This report ia accepted for publication in FIBRE SCIENCE AND TECHNOLOGY journal in 1982.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

composite materials, finite element analysis, exact solutions, laminates, shells, static analysis, various shell theories.

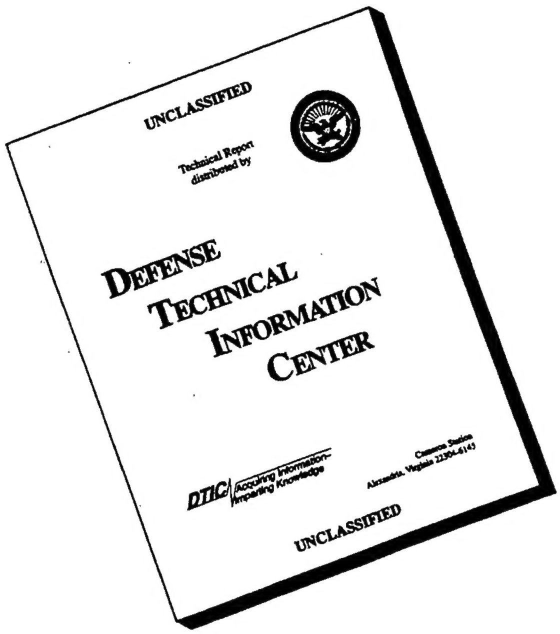
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A shear deformable finite element is developed based on generalized first approximation shell theories. Numerical results are presented for bending of layered, anisotropic, composite shells. Various shell theories (e.g., Sander's Love's, Donnell's, etc.) are included as special cases of the present element, and can be obtained by specifying the values of tracers. The finite element solutions are compared with the exact solutions for certain shell problems. The agreement is found to be very good.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DISCLAIMER NOTICE



THIS DOCUMENT IS BEST QUALITY AVAILABLE. THE COPY FURNISHED TO DTIC CONTAINED A SIGNIFICANT NUMBER OF PAGES WHICH DO NOT REPRODUCE LEGIBLY.

PB83-145409

COLLEGE OF ENGINEERING

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY

BLACKSBURG, VIRGINIA

The United States Air Force
AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
Structural Mechanics Program
Bolling AFB, D. C. 20332

Grant No. AFOSR-81-0142
Technical Report No. AFOSR-81-4

Report VPI-E-82.30

BENDING OF LAMINATED ANISOTROPIC SHELLS BY A SHEAR DEFORMABLE FINITE ELEMENT

J. N. Reddy
Professor

Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

Report VPI-E-82.30
October, 1982

ABSTRACT

A shear deformable finite element is developed based on generalized (i.e., account for shear deformations) first-approximation shell theories. Numerical results are presented for bending of layered, anisotropic, composite shells. Various shell theories (e.g., Sander's, Love's, Donnell's, etc.) are included as special cases of the present element, and can be obtained by giving numerical values to appropriate tracers. The present finite-element solutions are compared with the exact solutions for certain shell problems, and other solutions available in the literature. The agreement is found to be very good. Numerical results showing the effect of orientation of layers, boundary conditions, and material orthotropy on deflections are presented.

INTRODUCTION

Composite materials and reinforced plastics are increasingly used in automobiles, space vehicles, and pressure vessels. With the increased use of fiber-reinforced composites as structural elements, studies involving the thermomechanical behavior of composite-material shells are receiving considerable attention.

The first analysis that incorporated the bending-stretching coupling (due to unsymmetric lamination in composites) is due to Ambartsumyan [1,2]. In his analyses Ambartsumyan assumed that the individual orthotropic layers were oriented such that the principal axes of material symmetry coincided with the principal coordinates of the shell reference surface. Thus, Ambartsumyan's work dealt with what is now known as laminated orthotropic shells rather than laminated anisotropic shells;

in laminated anisotropic shells the individual layers are, in general, anisotropic and the principal axes of material symmetry of the individual layers do not coincide with the principal coordinates of the shell.

In 1962 Dong, Pister and Taylor [3] formulated a theory of thin shells laminated of anisotropic material. The theory is an extension of the theory developed by Stavsky [4] for laminated anisotropic plates to Donnell's shallow shell theory. Cheng and Ho [5] presented an analysis of laminated anisotropic cylindrical shells using Flügge's shell theory. A first approximation theory for the unsymmetric deformation of nonhomogeneous, anisotropic, elastic cylindrical shells was derived by Widera and Chung [6] by means of the asymptotic integration of the elasticity equations. For a homogeneous, isotropic material, the theory reduces to the Donnell equations. An exposition of various shell theories can be found in the article by Bert [7].

All of the theories discussed above are based on Kirchoff-Love's hypotheses in which the transverse shear deformation is neglected. Recent studies in layered anisotropic plates show (see the survey paper by the author [8]) that transverse shear deformation effects are more pronounced in composite plates and shells than in isotropic plates and shells. The effect of transverse shear deformation and transverse isotropy, as well as thermal expansion through the shell thickness were considered by Zukas and Vinson [9] and Dong and Tso [10]. The theory in [10] is only applicable to layered, orthotropic, cylindrical shells (i.e., the orthotropic axes of each layer coincide with the coordinate axes of the shell). Whitney and Sun [11] developed a shear deformable

theory for laminated cylindrical shells that includes both transverse shear deformation and transverse normal strain as well as expansional strains. Recently, Widera and Logan [12,13] presented refined theories for nonhomogeneous anisotropic cylindrical shells.

As far as the finite-element analysis of shells is concerned, layered composite shells have not received nearly as much attention as ordinary shells. The works of Dong [14] on statically-loaded orthotropic shell of revolution, Dong and Selna [15] on free vibration of the same, Wilson and Parsons [16] on static axisymmetric loading of arbitrarily thick orthotropic shells of revolution, and Schmit and Monforton [17] on laminated anisotropic cylindrical shells are the only ones that considered the finite element method before 1970's (note that the latter reference is the only one that considered laminated anisotropic shells). In 1970's there was an increased interest in the finite-element analysis of bending and vibration of laminated anisotropic shells. Apparently the first finite-element application in laminated anisotropic shells of arbitrary geometry is due to Thompson [18], who presented free (i.e., natural) vibration of general laminated anisotropic thin shells. Other finite-element analyses of layered anisotropic composite shells include the works of Panda and Natarajan [19], Shivakumar and Krishna Murty [20], Rao [21], and Siede and Chang [28].

The present paper is concerned with the development and application of a shear deformable shell element for the bending analysis of layered anisotropic shells. The shear-deformable theories used herein are generalizations of various classical shell theories, which are believed to be adequate for the prediction of overall response of layered composite shells. In the present investigation, several first-order

shell theories (Sanders, Love's, Loo's, Morley's and Donnell's) are considered and transverse shear strains are accounted for. For certain boundary conditions, lamination scheme and loading, exact form of the spatial variation of solutions are are obtained and are compared with the finite-element solutions.

GOVERNING EQUATIONS

Consider a shell constructed of a finite number of homogeneous, uniform thickness layers of an orthotropic material. Let the x-y surface coincide with the lines of the principal curvature of the midsurface of the shell, with z-axis normal (positive outward) to the midsurface of the shell (see Fig. 1). It is assumed that all of the layers in the shell remain elastic during the deformation, the Generalized Hooke's law is valid, and that no slip occurs between any two layers.

The displacement field in the shell is assumed to be of the form

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\,\psi_x(x,y) \\ v(x,y,z) &= v_0(x,y) + z\,\psi_y(x,y) \\ w(x,y,z) &= w_0(x,y) + z\,\psi_z(x,y) \end{aligned} \tag{1}$$

where u, v and w are the total displacements along x, y and z-directions, respectively; u_0, v_0 and w_0 are the displacements of a point in the coordinate surface along x, y and z coordinates; ψ_x, ψ_y and ψ_z are the bending rotations. The displacement field in eqn. (1) can be replaced by one containing the higher order terms in z (see, for example, Whitney and Sun [11]). However, the improvement one obtains in the prediction of the transverse shear deformation is at the expense of computational time (several additional dependent unknowns are introduced in the higher-order theories).

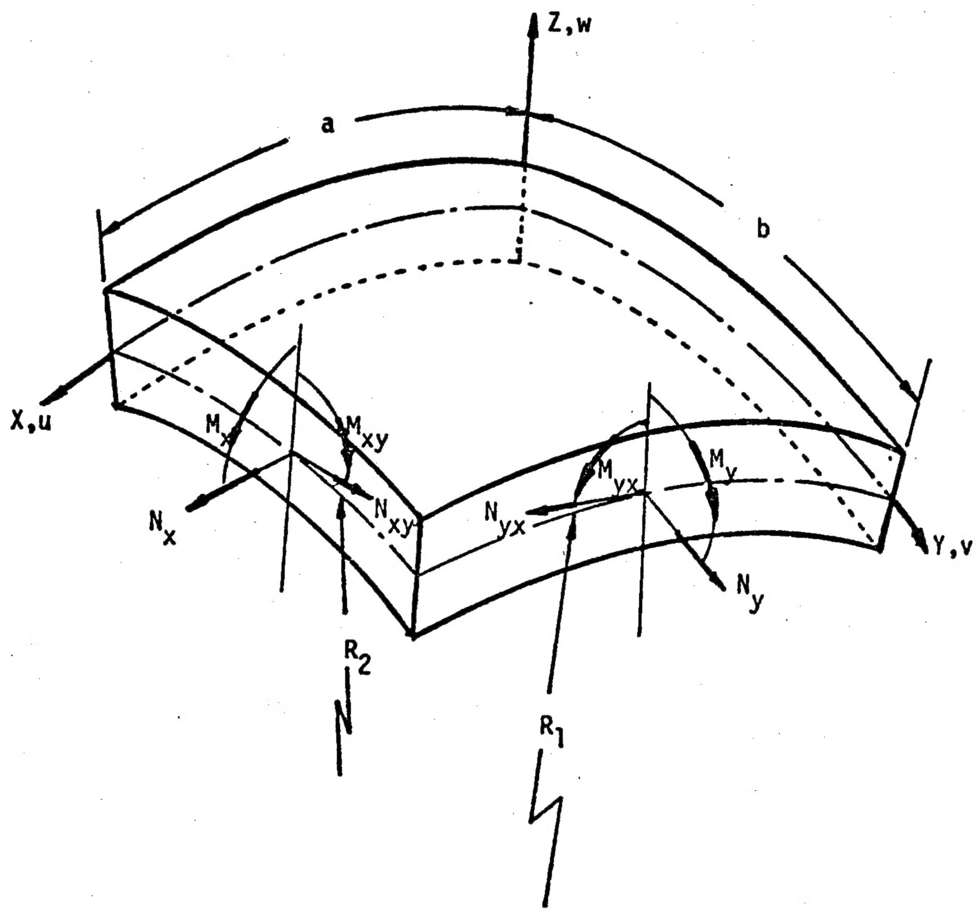


Figure 1. Differential element of a doubly curved shell.

Strain-displacement relationships for shells are very complex and a detailed derivation cannot be attempted in a paper of this type. References to various shell theories are made in Table 1. The strain-displacement relations for large- rotation (small strains) theory of shells can be simplified to the form

$$\varepsilon_i = \varepsilon_i^o + z\kappa_i \quad , \quad (i = 1,2,4,5,6), \tag{2}$$

where

$$\varepsilon_1^o = \frac{\partial u_o}{\partial x} + \frac{w_o}{R_1} + \underline{\frac{1}{2}\left(\frac{\partial w_o}{\partial x} + \frac{u_o}{R_1}\right)^2},$$

$$\varepsilon_2^o = \frac{\partial v_o}{\partial y} + \frac{w_o}{R_2} + \underline{\frac{1}{2}\left(\frac{\partial w_o}{\partial y} + \frac{v_o}{R_2}\right)^2},$$

$$\varepsilon_6^o = \frac{\partial u_o}{\partial y} + \frac{\partial v_o}{\partial x} + \underline{\left(\frac{\partial w_o}{\partial x} + \frac{u_o}{R_1}\right)\left(\frac{\partial w_o}{\partial y} + \frac{v_o}{R_2}\right)},$$

$$\varepsilon_4^o = \psi_y + \frac{\partial w_o}{\partial y} - C_1 \frac{v_o}{R_2},$$

$$\varepsilon_5^o = \psi_x + \frac{\partial w_o}{\partial x} - C_1 \frac{u_o}{R_1},$$

$$\kappa_1 = \frac{\partial \psi_x}{\partial x} + \frac{\psi_z}{R_1} + \underline{\frac{1}{R_1}\left(\frac{\partial u_o}{\partial x} + \frac{w_o}{R_1}\right)},$$

$$\kappa_2 = \frac{\partial \psi_y}{\partial y} + \frac{\psi_z}{R_2} + \underline{\frac{1}{R_2}\left(\frac{\partial v_o}{\partial y} + \frac{w_o}{R_2}\right)},$$

$$\kappa_6 = \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} + \underline{C_2\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\left(\frac{\partial v_o}{\partial x} - \frac{\partial u_o}{\partial y}\right)},$$

$$\kappa_4 = \kappa_5 = 0. \tag{3}$$

The underlined terms correspond to large rotations. The constants C_1

and C_2 are shell-theory tracers to be defined shortly (i.e., different values of C_i trace different shell theories in the literature), and R_1 and R_2 are the principal radii of curvature of the shell. The transverse normal strain ε_{33} is given by

$$\varepsilon_{33} = \psi_z + \frac{1}{2} (\psi_x^2 + \psi_y^2). \tag{4}$$

In the present study we make the assumption that $\varepsilon_{33} \simeq 0$, or equivalently,

$$\psi_z = - \frac{1}{2}(\psi_x^2 + \psi_y^2).$$

However, note that, the transverse shear strains are accounted for in the present theory. We also assume that u_i/R_1 and u_i/R_2 (for $u_i = u_o, v_o$), w_o/R_i^2 and ψ_z/R_i are small compared to the other quantities in ε_i ($i = 1,2,6$) and are neglected in the subsequent equations.

In the absence of body forces and moments, the equilibrium equations associated with a layered anisotropic shell can be simplified to the form (consistent with the assumptions made above),

$$\begin{aligned}
&N_{1,x} + N_{6,y} + Q_1(C_1/R_1) + [(C_4-C_3)/R_1 - C_2/R_2]M_{6,y} = 0, \\
&N_{6,x} + N_{2,y} + Q_2(C_2/R_2) + [(C_4-C_3)/R_2 - C_2/R_1]M_{6,x} = 0, \\
&Q_{1,x} + Q_{2,y} - N_1/R_1 - N_2/R_2 + N(w,N_i,M_i) = P, \\
&M_{1,x} + M_{6,y} - Q_1 = 0 \ , \\
&M_{6,x} + M_{2,y} - Q_2 = 0.
\end{aligned} \tag{5}$$

where C_i are the shell-theory tracers given in Table 1 and N_i and M_i (i = 1,2,6) are the stress and moment resultants defined in the customary way for a first-approximation shell theory ($z/R_i << 1$) as

$$(N_i, M_i) = \int_{-h/2}^{h/2} (1, z)\sigma_i \, dz \tag{6}$$

where h is the total laminate thickness, and σ_i are the stress components. The shear stress resultants Q_i are defined analogously:

$$(Q_1, Q_2) = \int_{-h/2}^{h/2} (\sigma_5, \sigma_4) dz. \tag{7}$$

The nonlinear operator $\mathcal{N}(\cdot)$ is defined by

$$\begin{aligned} \mathcal{N}(N_i, w) = {} & \frac{\partial}{\partial x} [(N_1 - \frac{M_1}{R_1})\frac{\partial w_o}{\partial x} + (N_6 - \frac{M_6}{R_2})\frac{\partial w_o}{\partial y}] \\ & + \frac{\partial}{\partial y} [(N_2 - \frac{M_2}{R_2})\frac{\partial w_o}{\partial y} + (N_6 - \frac{M_6}{R_1})\frac{\partial w_o}{\partial x}] \end{aligned} \tag{8}$$

Table 1 List of shell-theory tracers

Theory	C_1	C_2	C_3	C_4
Sanders [24]	1.0	0.5	1.0	1.5
Love's 1st Approximation [25]	1.0	0.0	1.0	1.0
Loo's Approximation [26]	0.0	0.0	1.0	1.0
Morley [27]	0.0	0.0	1.0	0.0
Donnell [28]	0.0	0.0	0.0	0.0

The thermoelastic constitutive equations for a layered anisotropic shell can be derived using the constitutive equations of individual layers (in much the same way as for plates; see, Reddy [8]). The equations are given by

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} A_{55} & A_{45} \\ A_{45} & A_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix},$$

$$\begin{Bmatrix} N_i \\ M_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ji} & D_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_i \\ \kappa_i \end{Bmatrix} - \begin{Bmatrix} N_i^T \\ M_i^T \end{Bmatrix}, \qquad (9)$$

where the stiffness coefficients A_{ij}, B_{ij} and D_{ij} are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1,z,z^2)dz, \quad (i,j = 1,2,6), \qquad (10)$$

$$A_{ij} = K^2 \int_{-h/2}^{h/2} Q_{ij}\, dz \quad , \quad (i,j = 4,5),$$

and the thermoelastic stress resultants and stress couples are defined by

$$(N_i^T, M_i^T) = \int_{-h/2}^{h/2} (1,z)Q_{ij}\alpha_j\, dz. \qquad (11)$$

Here α_j denotes the thermal expansion coefficients and Q_{ij} denote the plane-stress-reduced stiffnesses for individual layers. Since Q_{ij} are different for different layers, expressions in eqns. (9) and (10) should be integrated in parts, for example,

$$A_{ij} = K^2 \sum_{\ell=1}^{m} \int_{z_\ell}^{z_{\ell+1}} Q_{ij}^{(\ell)} dz \qquad (12)$$

where m is the total number of layers in the shell, and K is the shear correction coefficient.

Equations (2), (5), and (9) completely describe the equilibrium equations of a layered anisotropic shell. These equations cannot be solved in an exact form except for the small-displacement theory, simple-

supported boundary conditions, and sinusoidal distribution of the transverse load and thermal loads (see Hsu, Reddy and Bert [23]). In order to solve practically important problems that involve complex loadings and geometry, one must consider approximate methods of analysis. In the next section, a finite-element formulation is presented for the equations described in this section.

FINITE-ELEMENT FORMULATION

In the interest of brevity, we omit the algebraic details and describe briefly the finite-element model developed herein. The derivation proceeds along the same lines as described for layered anisotropic plates in [29]. Consider a finite element analog of the midsurface R of the shell. In a typical (finite) element R^e of the mesh, the generalized displacements u_o, v_o, w_o, ψ_x and ψ_y are interpolated by expressions of the form,

$$u_o = \sum_i^n u_{oi}\phi_i \quad , \quad v_o = \sum_i^n v_{oi}\phi_i, \text{ etc.} \tag{13}$$

where u_{oi} is the value of u_o at node i of the element R^e, ϕ_i is the finite-element interpolation function associated with node i, and n is the total number of nodes in the element. Here we have employed the same interpolation functions for each of the dependent variables. Substituting the expressions (13) into the governing equations (5), multiplying each equation by ϕ_i, and setting the integral of the result over R^e to zero (Galerkin integrals), we obtain (after carrying integration by parts of certain terms to relax the continuity on ϕ_i) the following form of element equations:

$$[K]\{\Delta\} = \{F\}. \tag{14}$$

Here K_{ij} denotes the stiffness matrix (5n by 5n), $\{\Delta\} = \{u_{oi}, v_{oi}, w_{oi}, \psi_{xi}, \psi_{yi}\}^T$, and $\{F\}$ is the vector of generalized forces.

In the present study the four-node, eight-node and nine-node rectangular isoparametric elements were employed. Analogous to the shear deformable theory of layered composite plates, the present theory can be viewed as a shear deformable theory derived from the classical shell theory by treating the slope-deflection relations (ε_4 and $\varepsilon_5 = 0$) as constraints, and including the constraints into the variational formulation of the shell equations by the penalty function method (see Reddy [29]). The elements derived using such theory are very stiff for thin shells, but yield good results for moderately thick shells. To overcome the difficulty, the so-called reduced integration technique must be employed in the evaluation of the stiffness coefficients associated with the shear energy terms (i.e., penalty terms). More specifically, the 1x1 Gauss rule must be used for shear energy terms and the standard 2x2 Gauss rule must be used for the bending terms when the four-node linear isoparametric element is used.

NUMERICAL RESULTS

All of the numerical results presented herein were obtained on an IBM 370/158 computer using the double precision arithmatics. Whenever biaxial symmetry existed in the problem, only one quadrant of the shell was modeled. Numerical results are presented using the linear and nonlinear theory. In the nonlinear analysis, a direct iteration method was used. The iteration begins with zero solution vector (so that at the end of the first iteration the linear solution is obtained) and

computes the linear element matrices. The solution at the end of the first iteration is used to recompute the element stiffness matrices (for the same load step) for the next iteration. This procedure is continued until the difference between two consecutive solution vectors, for a given load, differ by one percent. A shear correction factor of K^2 = 5/6 is used in all of the cases, and only one quadrant of the shell is modeled (whenever the biaxial symmetry existed).

To assess the numerical accuracy of the present model with respect to the element type (i.e., linear and quadratic elements), mesh (L2 = 2 by 2 mesh of the linear elements, Q2 = 2 by 2 mesh of the eight-node quadratic elements, and 9 = nine-node elements), and integration (F = full integration, R = reduced integration on all terms), several numerical experiments were carried using the Sanders shell-theory, and the results are listed in Table 2. The following nondimensionalizations are used:

$$\bar{w} = \frac{wE_2h^3}{P_oa^4} \times 10^2 \quad , \quad \bar{\sigma}_i = \frac{\sigma_i h^2}{P_oa^2} \tag{15}$$

Table 2 contains the nondimensionalized linear deflection and stresses for a four-layer (0°/90°/ 90°/0°), equal thickness, spherical shell ($R_1 = R_2 \equiv R$, $a/h = 10$) of material 1 ($G_{12} = G_{13}$, $\nu_{12} = \nu_{13}$):

$$\text{material 1:} \quad E_1/E_2 = 25,\ G_{12}/E_2 = 0.5,$$
$$G_{23}/E_2 = 0.2,\ \nu_{12} = 0.25. \tag{16}$$

The shell is subjected to sinusoidal distribution of normal loading,

$$P = P_o \sin\frac{\pi x}{a} \sin\frac{\pi y}{b}\ , \tag{17}$$

where a and b are the dimensions shown in Fig. 1. The following boundary conditions were imposed:

Table 2. Effect of reduced integration on the maximum deflection and stresses of a four-layer (0°/90°/90°/0°) spherical shell subjected to sinusoidal loading (material 1, a/b = 1.0, a/h = 10.0).

$$\bar{w} = w \frac{10^2 E_2 h^3}{P_o a^4} \qquad \bar{\sigma}_i = \sigma_i \frac{h^2}{P_o a^2}$$

R/h	Analysis		$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$\bar{\tau}_{yz}$	$\bar{\tau}_{xz}$
10	CFS		0.3229	0.1678	0.1032	0.04055	0.005660	0.01824
	FEM	Q4-R	0.3229	0.1678	0.1032	0.04055	0.005660	0.01824
		Q4-F	0.3228	0.1678	0.1031	0.04053	0.005655	0.01823
		Q2-R	0.3226	0.1643	0.1011	0.03964	0.005542	0.01785
		Q2-F	0.3223	0.1632	0.1000	0.03944	0.005466	0.01771
		Q2-R9	0.3235	0.1642	0.1010	0.03964	0.005539	0.01785
		Q2-F9	0.3241	0.1632	0.1000	0.03947	0.005467	0.01772
		L4-R	0.3291	0.1633	0.1019	0.03847	0.005582	0.01787
		L4-F	0.2939	0.1445	0.08971	0.03451	0.005417	0.01617
		L2-R	0.3479	0.1461	0.09538	0.03286	0.003219	0.01638
		L2-F	0.2337	0.1459	0.1165	0.02184	0.004658	0.01153
20	CFS		0.5254	0.3426	0.2331	0.04295	0.009930	0.03201
	FEM	Q4-R	0.5254	0.3426	0.2330	0.04294	0.009928	0.03201
		Q4-F	0.5253	0.3425	0.2329	0.04294	0.009928	0.03200
		Q2-R	0.5246	0.3350	0.2280	0.04197	0.009718	0.03131
		Q2-R9	0.5260	0.3350	0.2279	0.04197	0.009713	0.03131
		L4-R	0.5281	0.3268	0.2244	0.04058	0.009637	0.03087
		L4-F	0.4963	0.3047	0.2084	0.03798	0.009778	0.02934
		L2-R	0.5351	0.3359	0.2617	0.03330	0.008584	0.02695
		L2-F	0.4276	0.2597	0.1994	0.02615	0.008948	0.02245
50	CFS		0.6362	0.4541	0.3216	0.03474	0.01227	0.03955
	FEM	Q4-R	0.6361	0.4540	0.3215	0.03473	0.01227	0.03955
		Q4-F	0.6360	0.4539	0.3214	0.03473	0.01227	0.03955
		Q2-R	0.6351	0.4438	0.3144	0.3394	0.01200	0.03868
		Q2-F	0.6340	0.4427	0.3128	0.3389	0.01202	0.03865
		Q2-R9	0.6367	0.4438	0.3144	0.03394	0.01200	0.03867
		Q2-F9	0.6357	0.4427	0.3128	0.03391	0.01202	0.03865
		L4-R	0.6346	0.4290	0.3063	0.03271	0.01181	0.03783
		L4-F	0.6137	0.4118	0.2928	0.03144	0.01231	0.03699
100	CFS		0.6559	0.4797	0.3437	0.02979	0.01269	0.04090
	FEM	Q4-R	0.6558	0.4796	0.3437	0.02978	0.01268	0.04089
		Q4-F	0.6557	0.4796	0.3436	0.03978	0.01268	0.04089
		Q2-R	0.6547	0.4688	0.3360	0.02911	0.01241	0.03999
		Q2-F	0.6536	0.4678	0.3344	0.02907	0.01243	0.03997
		Q2-R9	0.6564	0.4688	0.3360	0.02911	0.01240	0.03998
		Q2-F9	0.6554	0.4678	0.3344	0.02909	0.01243	0.03997
		L4-R	0.6534	0.4524	0.3266	0.02809	0.01219	0.03906
		L4-F	0.6352	0.4365	0.3339	0.02710	0.01277	0.03839
		L2-R	0.6452	0.3836	0.2892	0.02294	0.01057	0.03317
		L2-F	0.5813	0.3349	0.2484	0.02000	0.01235	0.03111

Table 2. (Cont.)

R/h	Analysis		$\overline{w}$	$\overline{\sigma}_x$	$\overline{\sigma}_y$	$\overline{\tau}_{xy}$	$\overline{\tau}_{yz}$	$\overline{\tau}_{xz}$
∞ Plate	CFS		0.6627	0.4995	0.3589	0.02397	0.01283	0.04136
	FEM	Q4-R	0.6627	0.4954	0.3589	0.02396	0.01283	0.04136
		Q4-F	0.6626	0.4953	0.3588	0.02396	0.01283	0.04136
		Q2-R	0.6615	0.4842	0.3509	0.02342	0.01255	0.04044
		Q2-F	0.6605	0.4831	0.3492	0.02339	0.01258	0.04043
		Q2-R9	0.6633	0.4841	0.3508	0.02342	0.01254	0.04044
		Q2-F9	0.6623	0.4832	0.3492	0.02340	0.01257	0.04043
		L4-R	0.6599	0.4668	0.3406	0.02268	0.01233	0.03949
		L4-F	0.6427	0.4512	0.3280	0.02187	0.01097	0.03888
		L2-R	0.6508	0.3799	0.2838	0.01892	0.01067	0.03349
		L2-F	0.5901	0.3339	0.2454	0.01629	0.01225	0.03161

$$\text{SS-1:}\quad \begin{matrix} \underline{\text{natural}}: \begin{cases} N_1 = M_1 = 0 \text{ at } x = 0,a \\ N_2 = M_2 = 0 \text{ at } y = 0,b \end{cases} \\ \underline{\text{essential}}: \begin{cases} w_0 = v_0 = 0 \text{ at } x = 0,a \\ w_0 = u_0 = 0 \text{ at } y = 0,b \end{cases} \end{matrix} \qquad (18)$$

For this special case of boundary conditions, loading and cross-ply construction, one can obtain the exact form of the solution; see Hsu, Reddy and Bert [23].

From Table 2 one can conclude that the reduced integration improves the deflections as well as stresses for thin shells, when coarse meshes and lower order (i.e., linear) elements are used. For quadratic elements the reduced integration has only little effect even for a/h = 100. Numerical convergence of the element is apparent from an inspection of the deflections and stresses in Table 2. The agreement between the present finite element results and the exact solution is gratifyingly close. It should be pointed out that both the exact solution and the finite-element solutions are obtained for the same shell theories. The subsequent results were obtained using the reduced integration technique.

Next, the effect of thickness and type of shell theory on linear deflections and stresses of a four-layer (0°/90°/90°/0°, material 1) cylindrical shell ($R_1 = R$, $R_2 = 10^{30}$) subjected to sinusoidal distribution of normal force and simply supported (SS-1) boundary conditions is investigated (see Table 3). From the results (obtained using 2x2Q) of Table 3 it is clear that for shallow shells (R/h = 100) the difference between various theories is negligibly small for the radius-to-thickness ratio considered. For radius-to-thickness ratio of 10, the difference between the deflections obtained by various shell theories is noticeable. In the examples to follow, the Sanders shell theory was used.

Table 3 Dimensionless Deflections and Stresses for 4-Layer Cross-Ply (0°/90°/90°/0°) Freely Supported Cylindrical Shells Under Sinusoidal Loading by Different Shell Theories (material 1, a/b = 1, a/h = 10).

$$\bar{w} = w\,\frac{10^2 E_2 h^3}{P_0\, a^4}, \qquad \bar{\sigma}_i = \sigma_i \frac{h^2}{P_o a^2}$$

R/h	Theory	Source	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$	$-\bar{\tau}_{yz}$	$-\bar{\tau}_{xz}$
100	Donnell	CFS	0.6609	0.4998	0.3637	0.02694	0.01279	0.04124
		FEM	0.6606	0.4997	0.3636	0.02694	0.01279	0.04123
	Morley	CFS	0.6608	0.4999	0.3634	0.02696	0.01279	0.04124
		FEM	0.6608	0.4998	0.3634	0.03696	0.01279	0.04123
	Loo's Approx.	CFS	0.6608	0.4999	0.3633	0.03696	0.01279	0.04125
		FEM	0.6608	0.4995	0.3613	0.02696	0.01278	0.04124
	Love 1st Approx.	CFS	0.6610	0.5000	0.3630	0.02695	0.01278	0.04125
		FEM	0.6609	0.4999	0.3630	0.02695	0.01278	0.04125
	Sanders	CFS	0.6610	0.5001	0.3630	0.02694	0.01278	0.04126
		FEM	0.6609	0.5000	0.3630	0.02694	0.01278	0.04125
10	Donnell	CFS	0.5066	0.4233	0.3190	0.04172	0.009807	0.03162
		FEM	0.5066	0.4233	0.3189	0.4171	0.009806	0.03162
	Morley	CFS	0.5135	0.4287	0.3205	0.04233	0.009940	0.03212
		FEM	0.5135	0.4286	0.3205	0.04233	0.009939	0.03211
	Loo's Approx.	CFS	0.5157	0.4311	0.3213	0.04253	0.009984	0.03219
		FEM	0.5156	0.4310	0.3211	0.04252	0.009983	0.03218
	Love 1st Approx.	CFS	0.5244	0.4385	0.2990	0.04213	0.009249	0.03264
		FEM	0.5243	0.4384	0.2989	0.04213	0.009247	0.03263
	Sanders	CFS	0.5259	0.4403	0.2998	0.04162	0.009224	0.03270
		FEM	0.5259	0.4403	0.2997	0.04161	0.009223	0.03269

Table 4 shows the effect of number of layers, orientation of layers, and radius-to-thickness ratio on the nondimensionalized deflection of a simply supported (SS-1), angle-ply ($\theta/-\theta/+/-\ldots$) spherical shell ($R_1 = R_2 \equiv R$) under sinusoidal loading. The shell is constructed of material 2 ($G_{12} = G_{13}$, $\nu_{12}=\nu_{13}$):

$$\text{material 2:} \quad E_1/E_2 = 40, \; G_{12}/E_2 = 0.6, \; G_{23}/E_2 = 0.5, \; \nu_{12} = 0.25. \tag{19}$$

The finite-element solution differ noticeably from the exact solution (Fig. 2) for small number of layers (the difference decreases with increasing number of layers) and large values of the angle, θ. This error is due to the fact that the exact solution given in [23] satisfies different symmetry conditions than those imposed in the finite element method. This difference is directly proportional to stiffness coefficient B_{16}, whose magnitude is inversely proportional to the number of layers and directly proportional to $\theta (0 \leq \theta \leq 45°)$.

Figure 3 shows the effect of aspect ratio (b/a) and radius-to-thickness ratio (R/h) on the nondimensional deflections ($\bar{w}_t = wh/\alpha_1 T_o a^2$) for simply supported (SS-1) spherical shell ($R_1 = R_2 \equiv R$, material 1) under simusoidal-thermal loading.

$$T = T_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}. \tag{20}$$

It is clear from the figure that thickness shear deformation has a pronounced effect on the nondimensionalized deflection, $\bar{w}_t$; the effect decreases with increasing number of layers (the two-layer solution forms the upper bound while the single-layer solution forms the lower bound).

Table 4. Nondimensional deflection as a function of number of layers, angles, and radius-to-thickness ratio for freely supported angle-ply ($\theta/-\theta/\ldots$) spherical shell ($a/b = 1$, $a/h = 10$, material 2).

$$\bar{w} = w\frac{10^2 E_2 h^3}{P_o a^4}$$

R/h	Source	θ = 5°			θ = 30°			θ = 45°		
		n=2	N=4	N=16	n=2	n=4	n=16	n=2	n=4	n=16
5	CFS	0.1182	0.1182	0.1182	0.03165	0.03165	0.03165	0.01183	0.01183	0.01183
	FEM	0.1424	0.1255	0.1194	0.03509	0.03147	0.03140	0.009382	0.01079	0.01138
10	CFS	0.2673	0.2673	0.2673	0.09552	0.09552	0.09552	0.04170	0.04170	0.04170
	FEM	0.3436	0.2924	0.2720	0.1306	0.1002	0.09566	0.05215	0.04061	0.04079
20	CFS	0.3820	0.3820	0.3820	0.1908	0.1908	0.1908	0.1127	0.1127	0.1127
	FEM	0.4715	0.4112	0.3873	0.3089	0.2070	0.1920	0.1783	0.1175	0.1118
30	CFS	0.4146	0.4146	0.4146	0.2339	0.2339	0.2339	0.1646	0.1646	0.1646
	FEM	0.4898	0.4400	0.4189	0.4038	0.2562	0.2354	0.2825	0.1753	0.1640
40	CFS	0.4273	0.4273	0.2540	0.2540	0.2540	0.2540	0.1962	0.1962	0.1962
	FEM	0.4953	0.4496	0.4308	0.4505	0.2791	0.2556	0.3507	0.2110	0.1960
50	CFS	0.4334	0.4334	0.4334	0.2645	0.2645	0.2645	0.2153	0.2153	0.2153
	FEM	0.4989	0.4535	0.4365	0.4752	0.2910	0.2661	0.3940	0.2329	0.2154
60	CFS	0.4369	0.4369	0.4369	0.2706	0.2706	0.2706	0.2274	0.2274	0.2274
	FEM	0.5003	0.4553	0.4395	0.4895	0.2979	0.2722	0.4223	0.2468	0.2276
80	CFS	0.4403	0.4403	0.4403	0.2769	0.2769	0.2769	0.2408	0.2408	0.2408
	FEM	0.5008	0.4567	0.4424	0.5039	0.3049	0.2786	0.4550	0.2626	0.2413
100	CFS	0.4419	0.4419	0.4419	0.2799	0.2799	0.2799	0.2476	0.2476	0.2476
	FEM	0.5016	0.4570	0.4437	0.5106	0.3082	0.2815	0.4722	0.2707	0.2483
∞	CFS	0.4448	0.4448	0.4448	0.2355	0.2855	0.2855	0.2605	0.2605	0.2605
(plate)	FEM	0.5019	0.4546	0.4454	0.5199	0.3137	0.2870	0.5119	0.2879	0.2620

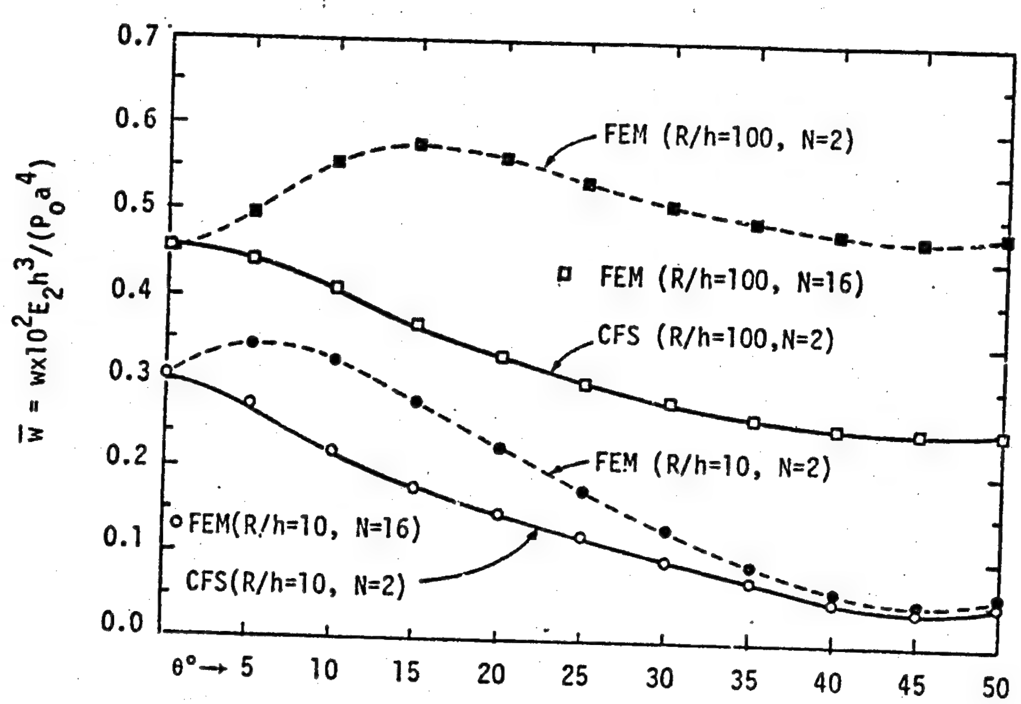
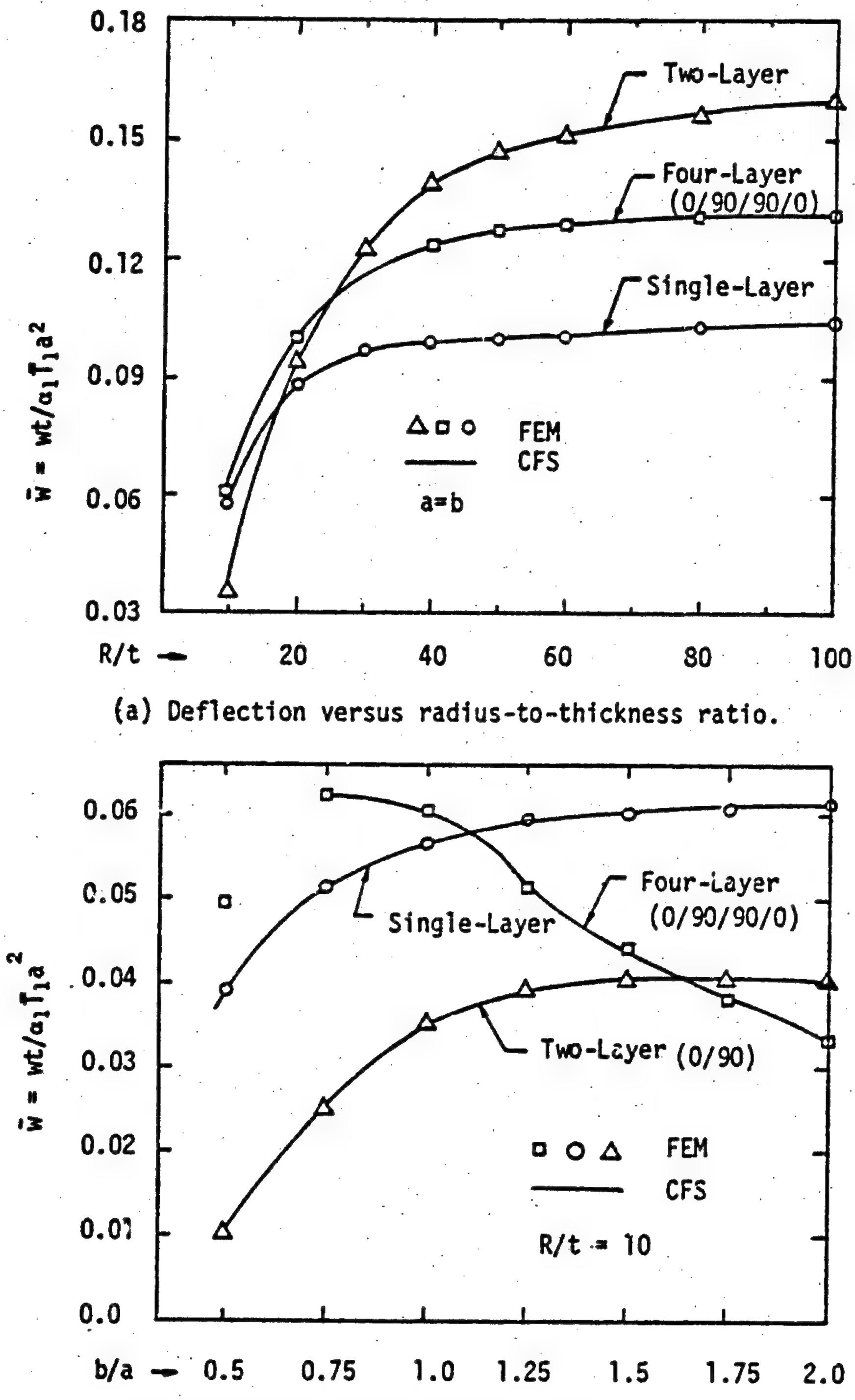


Figure 2. Effect of lamination scheme and radius-to-thickness ratio on the nondimensionalized deflection of a sperical shell under sinusoidal transverse loading (SS-1, a/h = 10, material 2)



(a) Deflection versus radius-to-thickness ratio.

(b) Deflection versus aspect ratio.

Figure 3. Effect of thickness and aspect ratio on the deflection of cross-ply doubly-curved shells under thermal loading. (material 1, $R_1 = R_2 = R$)

The natural vibration of layered composite shells was also investigated using the element presented herein. Tables 5 and 6 contain the values of the nondimensionalized fundamental frequencies of doubly curved (spherical) shells of cross-ply and angle-ply construction, respectively. Note that as the number of layers is increased, the frequencies do not change appreciably (for the same total thickness of the shell). The nondimensionalized frequencies of the cross-ply shells are about half of those of the angle-ply shells.

Table 5. Effect of radius-to-thickness ratio on the dimensionless fundamental frequency, $\lambda = \omega R \sqrt{\rho/E_2}$, of cross-ply spherical shells† ($R_1 = R_2 = R$, material 2, $a/b = 1$, $a/h = 10$).

R/ h	Source	0°/90°	0°/90°/0°	0°/90°/90°/0°
5	CFS	1.0536	1.0739	1.0757
	FEM	1.0538	1.0746	1.0718
10	CFS	1.4452	1.6085	1.6146
	FEM	1.4442	1.6128	1.6167
20	CFS	2.1484	2.675	2.688
	FEM	2.1518	2.6818	2.6812
30	CFS	2.934	3.8184	3.8379
	FEM	2.9352	3.8175	3.8073
40	CFS	3.7636	4.9944	5.0204
	FEM	3.7538	4.9824	5.0220
50	CFS	4.6151	6.186	6.218
	FEM	4.6035	6.0045	6.2695
60	CFS	5.4786	7.3854	7.4238
	FEM	5.4592	7.3926	7.5222
80	CFS	7.2247	9.7960	9.9480
	FEM	7.1954	9.8280	9.8472
100	CFS	8.9461	12.225	12.274
	FEM	8.9001	12.163	12.238

†freely-supported boundary conditions are used.

Table 6. Effect of radius-to-thickness ratio on the nondimensional fundamental frequencies of angle-ply spherical shells† ($R_1 = R_2 = R$, material 2, a/b = 1, a/h = 10).

R/h	Source	45°/-45°	45°/-45°/45°/-45°	45°/-45°/ n=8
5	CFS		2.2215	
	FEM	2.2229	2.2232	2.2236
10	CFS		4.4429	
	FEM	3.6286	4.4456	4.4461
20	CFS		6.0540	
	FEM	4.5238	5.7334	5.9716
30	CFS		7.6344	
	FEM	5.5908	7.2330	7.536
40	CFS		9.4032	
	FEM	6.8100	8.9036	9.2804
50	CFS		11.275	
	FEM	8.100	10,6895	11.1315
60	CFS		13.266	
	FEM	9.4794	12.516	13.0434
80	CFS		17.1688	
	FEM	12.2704	16.2752	16.9608
100	CFS		21.202	
	FEM	15.129	20.095	20.948

† Freely-supported boundary conditions are used.

SUMMARY AND CONCLUSIONS

A finite element based on a shear deformation theory of layered, anisotropic, composite shells is presented. The element contains as special cases generalized first-approximation shell theories of Love, Sanders, Donnell, Morley, and Loo. The generalization is to include the transverse shear strains. Numerical results are presented for cross-ply as well as angle-ply shells under various mechanical and thermal loadings. It is concluded that the effect of shear deformation on the deflection and stresses is significant, and that reduced integration improves the deflections and stresses for thin shells when coarse meshes and lower order elements are employed. Use of higher-order theories and extension of the present element to the nonlinear (geometrically) analysis of shells is far from complete. Although the nonlinear terms in the sense of von Karman are included in the formulation presented herein, no numerical results are included. These will appear elsewhere when the investigation is completed.

ACKNOWLEDGMENTS

The results reported in the paper were obtained during an investigation supported by Structural Mechanics Program of the Air Force Office of Scientific Research (Grant AFOSR-81-0142) and the Structures Research Section of NASA-Lewis (NAG. 3-208). The support is gratefully acknowledged. Thanks are also due to Mr. Y. S. Hsu for the computational assistance with certain results presented herein. It is a pleasure to acknowledge the skillfull typing of Mrs. Vanessa McCoy.

REFERENCES

1. Ambartsumyan, S. A., "Calculation of Laminated Anisotropic Shells," Izvestiia Akademiia Nauk Armenskoi SSR, Ser. Fiz. Mat. Est. Tekh. Nauk., Vol. 6, No. 3, 1953, p. 15.

2. Ambartsumyan, S. A., Theory of Anisotropic Shells, Moscow, 1961; English translation, NASA TT F-118, May 1964.

3. Dong, S. B., Pister, K. S. and Taylor, R. L., "On the Theory of Laminated Anisotropic Shells and Plates," Journal of Aerospace Sciences, Vol. 29, 1962, pp. 969-975.

4. Stavsky, Y., "Bending and Stretching of Laminated Anisotropic Plates," Journal of the Engineering Mechanics Division, Vol. 87, No. EM6, 1961, p. 31.

5. Cheng, S. and Ho, B. P. C., "Stability of Heterogeneous Aeolotropic Cylindrical Shells Under Combined Loading," American Institute of Aeronautics and Astronautics Journal, Vol. 1, No. 4, 1963, pp. 892-898.

6. Widera, O. E., and Chung, S. W., "A Theory for Non-Homogeneous Anisotropic Cylindrical Shells," Journal of Applied Mechanics (ZAMP), Vol. 21, 1970, pp. 378-399.

7. Bert, C. W., "Analysis of Shells," Analysis and Performance of Composites, Broutman, L. J. (ed.), Wiley, New York, 1980, Ch. 5, pp. 207-258.

8. Reddy, J. N., "Analysis of Layered Composite Plates Accounting Large Deflections and Transverse Shear Strains," Recent Advances in Nonlinear Computational Mechanics, E. Hinton, et al. (eds.), Pine Ridge Press, Swansea, United Kingdom, (to appear).

9. Zukas, J. A. and Vinson, J. R., "Laminated Transversely Isotropic Cylindrical Shells," Journal of Applied Mechanics, Vol. 1971, pp. 400-407.

10. Dong, S. B. and Tso, F. K. W., "On a Laminated Orthotropic Shell Theory Including Transverse Shear Deformation," Journal of Applied Mechanics, Vol. 39, 1972, pp. 1091-1097.

11. Whitney, J. M. and Sun, C. T., "A Refined Theory for Laminated Anisotropic, Cylindrical Shells," Journal of Applied Mechanics, Vol. 41, 1974, pp. 471-476.

12. Widera, G. E. O. and Logan, D. L., "Refined Theories for Nonhomogeneous Anisotropic Cylindrical Shells: Part I-Derivation," Journal of the Engineering Mechanics Division, Vol. 106, No. EM6, 1980, pp. 1053-1074.

13. Logan, D. L. and Widera, G. E. O., "Refined Theories for Nonhomogeneous Anisotropic Cylindrical Shells: Part II-Application," Journal of the Engineering Mechanics Division, Vol. 106, No. EM6, 1980, pp. 1075-1090.

14. Dong. S. B., "Analysis of Laminated Shells of Revolution," Journal of the Engineering Mechanics Division, Vol. 92, EM6, 1966, p. 135.

15. Dong, S. B. and Selna, L. G., "Natural Vibrations of Laminated Orthotropic Shells of Revolution," Journal of Composite Materials, Vol. 4, No. 1, 1970, pp. 2-19.

16. Wilson, E. A. and Parsons, B., "The Finite Element Analysis of Filament-Reinforced Axisymmetric Bodies," Fibre Science and Technology, Vol. 2, 1969, pp. 155-156.

17. Schmit, L. A. and Monforton, G. R., "Finite Element Analysis of Sandwich Plate and Laminate Shells with Laminated Faces," American Institute of Aeronautics and Astronautics Journal, Vol. 8, 1970, pp. 1454-1461.

18. Thompson, G. L., "Finite-Element Analysis for Free Vibration of General Anisotropic Laminated Thin Shells," Composite Materials in Engineering Design (Proc. 6th St. Louis Sympos., May 11-12, 1972), Noton, B. R. (ed.), American Society for Metals, Metals Park, Ohio, 1973.

19. Panda, S. C. and Natarajan, R., "Finite Element Analysis of Laminated Shells of Revolution," Computers and Structures, Vol. 6, 1976, pp. 61-64.

20. Shivakumar, K. N. and Krishna Murty, A. V., "A High Precision Ring Element for Vibrations of Laminated Shells," Journal of Sound and Vibration, Vol. 58, No. 3, 1978, pp. 311-318.

21. Rao, K. P., "A Rectangular Laminated Anisotropic Shallow Thin Shell Finite Element," Computer Methods in Applied Mechanics and Engineering, Vol. 15, 1978, pp. 13-33.

22. Siede, P. and Chang, P. H. H., "Finite Element Analysis of Laminated Plates and Shells," NASA CR-157106, 1978 (132 pages).

23. Hsu, Y. S., Reddy, J. N., and Bert, C. W., "Thermoelasticity of Circular Cylindrical Shells Laminated of Bimodulus Composite Materials," Journal of Thermal Stresses, Vol. 4, No. 2, 1981, pp. 115-177.

24. Sanders, Jr., J. L., "An Improved First Approximation Theory for Thin Shells," NASA TR R-24, June 1959.

25. Love, A. E. H., "On the Small Free Vibrations and Deformations of the Elastic Shells," Philosophical Transactions of the Royal Society (London), Ser. A, Vol. 17, 1888, pp. 491-546.

26. Loo, T. T., "An Extension of Donnell's Equation for a Circular Cylindrical Shell," Journal of Aeronautical Science, Vol. 24, 1957, pp. 390-391.

27. Morley, L. S. D., "An Improvement of Donnell's Approximation of Thin-Walled Circular Cylinders," Quarterly Journal of Mechanics and Applied Mathematics, Vol. 8, 1959, pp. 169-176.

28. Donnell, L. H., "Stability of Thin Walled Tubes in Torsion," NACA Report 479, 1933.

29. Reddy, J. N., "A Penalty Plate Bending Element for the Analysis of Laminated Anisotropic Composite Plates," International Journal for Numerical Methods in Engineering, Vol. 15, 1980, pp. 1187-1206.

UNITED STATES DEPARTMENT OF COMMERCE
National Technical Information Service
5285 Port Royal Road
Springfield, Virginia 22161
OFFICE OF THE DIRECTOR

NEW NTIS ABSTRACT NEWSLETTERS!

Dear Professional:

I wish I could tell you how much time and money the NTIS Abstract Newsletters could save you.

But, I can't. Because I don't know how much time and money you unknowingly spend on costly research that actually duplicates the work of others. Yet this research information is readily available to you when you know how to access it.

That's what NTIS Abstract Newsletters are all about.

<u>NTIS Abstract Newsletters Summarize 70,000 New Reports Annually</u>

You see, NTIS information specialists index, analyze, and categorize 70,000 new research documents a year. Each report is judged for its potential value for thousands of current subscribers. Then it is summarized in a brief abstract and published in one or more Newsletters. You receive only those abstracts of interest to your professional field.

NTIS exists to help you find--and provide you with--research results and information. You'll know what you need to know--when you need to know it--with NTIS Abstract Newsletters.

The Newsletters are a speedy, comprehensive, economical way for you to keep current on the latest research and to cut your risk of duplicating work already available in a report from NTIS.

Your Newsletters will suggest possible directions and starting points for your own research. Knowing research applications of others will help you get more out of every research dollar you invest.

Worldwide Sources

The NTIS collection also contains selected information on scientific and technical developments around the world.

NTIS is now actively expanding its sources of foreign technology to add to more than 300,000 foreign reports already on hand--including translated materials from the U.S.S.R., other Eastern European countries, Japan, and major Western countries.

Low Subscription Rate

> By now, you may well be apprehensive over the cost of this remarkable Newsletter service. If so, you will be pleasantly surprised to learn you can subscribe to the *Energy Abstract Newsletter,* for example, for about $1.83 a week!

NTIS is a unique self-supporting Government agency offering economical information products and services on a cost-recovery basis.

You Get Reports on Research Not Published By Any Other Source

NTIS Abstract Newsletters let you know about all Government-sponsored research in your field. Each week you'll learn about research not reported or available from any other source.

And when you spot a report you need, NTIS can provide it for you in a matter of days, usually for about $14 in paper copy or only $4.50 in microfiche. Your Newsletters contain a handy report order form for your convenience.

Annual Subject Index Included

Your subscription includes an Annual Subject Index to give permanent reference value to your Newsletter.

☆ U.S. GOVERNMENT PRINTING OFFICE: 1983 - 381-009 - 227/524

Of course, your subscription is tax deductible as a business expense in most instances.

More than that, when you consider the wasted time and costly mistakes the Newsletters can help you avoid, you will begin to appreciate what a practical investment NTIS Abstract Newsletters really are. For example:

- *MANUFACTURING TECHNOLOGY:* one of the newest Newsletters that meets industry's most pressing need right now...brings news of the latest research on robotics and CAD/CAM. $125*

- *FOREIGN TECHNOLOGY:* abstracts of latest technological and scientific research in all fields from Japan, United Kingdom, France, U.S.S.R., and countries all over the world. SPECIAL CHARTER RATE: $75*

- *ENVIRONMENTAL POLLUTION & CONTROL:* one of the most sought after Newsletters covering air, noise, solid wastes, water, pesticides, radiation, health and safety and environmental impact statements. $95*

- *ENERGY:* brings you a wealth of information on energy use; supply and demand; power and heat generation; energy policies, regulations and studies; engines and fuels, and much more. $95*

- *COMPUTERS, CONTROL, AND INFORMATION THEORY:* important information, and lots of it, concerning this, one of the fastest growing and most competitive industries worldwide. $95*

- *MATERIAL SCIENCES:* applicable to nearly every industry with abstracts touching on a huge variety of processes and products. $80*

- *MEDICINE & BIOLOGY:* covers an enormous range of information from anatomy to zoology. Every facet of medicine and biology is touched upon. $75*

And any one of the abstracts in these Newsletters may be the added element of success for your ongoing research, contract proposals, cost reduction efforts, and profits.

NTIS regularly publishes information not available elsewhere, and frequently brings you news of research before it appears in professional journals and magazines. You simply should not risk acting without benefit of complete, current information on what's happening in Government-sponsored research in your field.

*One-year subscription rate

(Over, please)

You need NTIS Abstract Newsletters and here's how to order them.

Complete the order form at the bottom of this letter and mail it back today to the address below. You may use any of the options listed in order to pay for your subscription.

You are guaranteed satisfaction. If your Newsletters do not satisfy you for any reason, you may cancel and receive a credit or refund for the balance of your subscriptions.

I invite you to subscribe today.

Sincerely,

Joseph F. Caponio

Joseph F. Caponio
Acting Director

NTIS 5285 Port Royal Road
Springfield, VA 22161 or Call (703) 487-4630

Please start my subscription to the following *Abstract Newsletters*:

Special Offer Only:

Foreign Technology ____________
– Save $15 or $30 –
Charter Rate: 2 yrs. $150 ☐ (Reg. Rate $180)
Charter Rate: 1 yr. $75 ☐ (Reg. Rate $90)
Additional subscriptions $25 each annually.

Name ____________
Occupation ____________
Company ____________
Address ____________
City / State/ZIP ____________

Other Titles:

Title ____________
☐ plus ____ additional subscriptions to this Newsletter $25 each
Title ____________
☐ plus ____ additional subscriptions to this Newsletter $25 each

☐ Charge to my Deposit Account No. ____________
☐ Here is my check to NTIS for $ ____________
☐ Bill me
Charge to ☐ American Express ☐ VISA ☐ MasterCard
Card No. ____________
Expires ____________

Cost
$ ____
$ ____
$ ____
$ ____
$ ____
TOTAL $ ____

____
Signature (required to validate order)

PR-733-3
4/83